Disc. 105-S28/From the May-June 2008 *ACI Structural Journal*, p. 290

Plastic Hinge Length of Reinforced Concrete Columns. Paper by Sungjin Bae and Oguzhan Bayrak

Discussion by N. Subramanian

Gaithersburg, MD

Plastic hinges form at the maximum moment region of reinforced concrete columns. The determination of the length of plastic hinge length is a critical step in predicting the lateral load-drift response of columns. As it is difficult to estimate the plastic hinge length by using sophisticated computer programs, it is often estimated based on experimental data or by using empirical equations. However, several factors influence the length of plastic hinge, such as: 1) level of axial load; 2) moment gradient; 3) the value of shear stress in the plastic hinge region; 4) the amount and mechanical properties of longitudinal and transverse reinforcement; 5) strength of concrete; and 6) level of confinement provided in the potential plastic hinge zone. The simplified equations available in literature do not contain all or most of the aforementioned factors. Hence, large variations exist in the value of plastic hinge length calculated using these empirical equations, as shown clearly by the authors in Fig. 2. With this background, the authors are to be commended for conducting new experimental work on the influence of shear-span ratio (L/h) and the effect of axial force on the plastic hinge length. They have also developed the following equation based on their research

$$\frac{l_p}{h} = \left[0.3\left(\frac{P}{P_o}\right) + 3\left(\frac{A_s}{A_g}\right) - 0.1\right]\left(\frac{L}{h}\right) + 0.25 \geq 0.25 \qquad (11)$$

The quantities in Eq. (11) are defined in the paper.

Compared to other equations presented by other researchers (Eq. (1) to (10) of the paper), this equation is comprehensive and includes the level of axial force (P/P_o), shear-span ratio (L/h), and amount of longitudinal steel (A_s/A_g). However, it does not consider the other parameters mentioned earlier, especially the strength of concrete and mechanical properties of reinforcements.

Interestingly, Berry et al.[26] contributed a paper in the same issue of the journal and presented the following equation for calculating the length of plastic hinge.

$$l_p = 0.05L + 0.1 f_y d_b / \sqrt{f_c'} \text{ (MPa)} \qquad (12a)$$

$$l_p = 0.05L + 0.008 f_y d_b / \sqrt{f_c'} \text{ (psi)} \qquad (12b)$$

The aforementioned equation does not consider the parameters considered by the authors does but includes the strength of concrete and properties and amount of longitudinal steel. Berry et al.[26] compared this equation with those proposed by Corley[5] and Paulay and Priestley[8] and found that it provides adequate estimates of the force displacement response as compared to the 37 experimental tests on large-scale circular bridge columns. Comparing the results of this equation with the measured values of given in the paper, however, it is found that it does not predict the plastic hinge length accurately, especially when P/P_o is greater than 0.2 (refer to Table A).

It has to be noted that the measured values of Table 2 of the paper do not match with those measured values given in Fig. 8 of the paper. Hence the values as per Fig. 8 only are compared in Table A. There seems to be some error in the calculations made as per the equations suggested by Paulay and Priestley.[8] Thus the values in Table 2 as per the equation suggested by Paulay and Priestley[8] should be read as $0.44h$, $0.59h$, $0.43h$, and $0.43h$ instead of $0.80h$, $0.96h$, $0.72h$, and $0.72h$. It may be recalled that Paulay and Priestley[8] reported that, for all practical columns, the value of plastic hinge length will be approximately $0.5h$.

It is interesting to note that Baker,[1] in his early work, considered most of the parameters affecting plastic hinge length, except amount and mechanical properties of longitudinal and transverse reinforcement. It is also found from Eq. (10) and (12) that the contribution of the longitudinal reinforcement effect is not considerable. Hence, the discusser would like to suggest the following equation, based on Baker's work and Fig. 10 of the author's paper.

$$\frac{l_p}{h} = 0.9\left[1 + 0.5\frac{P}{P_o}\right]k_1\left(\frac{L}{h}\right)^{0.25} \text{ (MPa) for } P/P_o > 0.2 \qquad (13a)$$

$$\frac{l_p}{h} = 0.25 \text{ for } P/P_o \leq 0.2 \qquad (13b)$$

where $k_1 = 0.6$ when $f_c' = 5100$ psi (35.2 MPa); and $k_1 = 0.9$ when $f_c' = 1700$ psi (11.7 MPa) and

$$\begin{aligned} k_1 &= 0.9 - (0.3/23.5)(f_c' - 11.7), \text{ where } 35.2 \text{ MPa} < f_c' < 11.7 \text{ MPa} \\ k_1 &= 0.9 - (0.3/3400)(f_c' - 1700), \text{ where } 5100 \text{ psi} < f_c' < 1700 \text{ psi} \end{aligned} \qquad (13c)$$

The above equations compare favorably with the measured values reported in the paper (refer to Table A).

The authors should be commended for identifying (based on their experiments) the fact that the required length l_o at the bottom of the column (over which closely-spaced reinforcement has to be provided) has to be increased from the present $1.0h$ (as per ACI 318-05) to a minimum of $1.5h$ (refer to Fig. 8 of the paper).

REFERENCES

26. Berry, M. P.; Lehman, D. E.; and Lowes, L. N., "Lumped-Plasticity Models for Performance Simulation of Bridge Columns," *ACI Structural Journal*, V. 105, No. 3, May-June 2008, pp. 270-279.

Table A—Predicted hinge lengths

Specimen	h, in. (mm)	f'_c, ksi (MPa)	d_b, in. (mm)	f_y, ksi (MPa)	P/P_o	ρ, %	L/h	Measured	Eq. (11)	Eq. (12)	Eq. (13)
S24-2UT	24 (610)	6.3 (43.4)	0.88 (22)	7.3 (50.3)	0.5	1.25	5	$0.95h$	$0.69h$	$0.28h$	$0.83h$
S17-3UT	17.25 (440)	6.3 (43.4)	0.63 (16)	7.2 (49.6)	0.5	1.25	7	$1.05h$	$0.86h$	$0.37h$	$0.9h$
S24-4UT	24 (610)	5.3 (36.5)	0.88 (22)	5.8 (40.0)	0.2	1.25	5	$0.25h$	$0.25h$	$0.27h$	$0.25h$
S24-5UT	24 (610)	6.0 (41.4)	0.88 (22)	5.8 (40.0)	0.2	1.25	5	$0.25h$	$0.25h$	$0.27h$	$0.25h$

Disc. 105-S28/From the May-June 2008 *ACI Structural Journal*, p. 290

Plastic Hinge Length of Reinforced Concrete Columns. Paper by Sungjin Bae and Oguzhan Bayrak

Discussion by Abdelsamie Elmenshawi

ACI member, PhD, Structural Engineer, Kassian Dyck & Associates, Calgary, AB, Canada

Predicting the length of a potential plastic hinge in a reinforced concrete element subjected to cyclic loading is required to evaluate the element performance. So, the research discussed an important issue in evaluating seismic performance, and many other researchers studied this issue, as presented in the paper. The discusser, however, wishes to highlight some issues regarding the influencing factor on the plastic hinge length and the presented "concrete compression strain method" as follows:

1. Most of the referenced literature was for reinforced concrete elements tested under monotonic loading.[1-5,10] Also, the method of concrete compression strain depended on a sectional analysis that disregarded the characteristics of cyclic loading. The results of monotonic tests do not consider the effect of residual tensile strain on hinge extension, moment-shear interaction, and reinforcement Bauschinger effect. Therefore, it is expected that the formulas depicted through monotonic tests will underestimate the length of potential plastic hinges (Table 2). Parameters affecting plastic hinge length under monotonic loading—that is, sectional depth, reinforcement ratio, and neutral axis depth—are not likely to affect the hinge length under cyclic loading.[7]

2. The paper is not clear on the significance of shear stresses and fixed-end rotation: sometimes neglected and sometimes not. It was previously confirmed that the presence of diagonal cracks increases the length of the plastic hinge.[11] In general, the impact of shear increases as the shear span-depth ratio (L/h) decreases; accordingly, the plastic hinge length will increase as the shear span-depth ratio decreases. Yet, in column tests, the shear spread may be decreased due to the presence of axial load, which improves the concrete shear strength and reduces shear deformation. Therefore, in beam, under cyclic loading, the length of a plastic hinge will significantly be affected by the shear spread. The disagreement in the paper may be ascribed to the fact that the chosen shear span-depth ratio is relatively high. The tested ratios were 5 and 7, which were enough to ensure flexural behavior under cyclic loading. Flexural behavior can be obtained under cyclic loading if the shear span-depth ratio exceeds 4.[27] Also, one test specimen is not enough to reach a confident conclusion. The effect of shear span-depth ratio on plastic hinge length was mainly performed analytically by the authors, where the analytical model has many cons, as will be discussed in the following.

3. The method of concrete compression strain mainly depended on relating the sectional failure to a critical value of concrete strain in compression. Although this statement is generally accepted, under cyclic loading, as the discusser believes, the matter is different, as there is no compression or tension side, because either side can be under tension or compression. The discusser believes that cover spalling and subsequent damage are due to the residual tensile strain, which inhibits the cracks formed in a previous tension half-cycle to close under a subsequent compression half-cycle. Therefore, plastic hinges form under cyclic loading due to the accumulated damage under increasing residual tensile strains.

4. To calculate the hinge length, the authors neglected the effect of reinforcement buckling because they were concerned with the ascending branch only in the moment-curvature relationship. Typically, it is known that the ascending branch in a load-deformation relationship terminates with yielding (either first yield or idealized) in tension. Beyond the yield point, a significant drop in the tangent stiffness can be noticed; however, the tangent stiffness will not change to a negative value until maximum lateral strength is reached; as a result, strength degradation commences. Therefore, the discusser agrees with the authors in that the plastic hinge can be assumed to start with the beginning of strength softening; nevertheless, the discusser disagrees with the authors in that the ascending branch is enough to anticipate the hinge formation because the ascending branch ends with the yield in tension. The ascending branch can represent only preyield deformation, where hinge is formed due to post-yield (plastic) deformation.

5. Although the main idea of the paper was to present an analytical tool to estimate plastic hinge length, the authors did not provide any clues about the material modeling and the nonlinear methodology being used except in Fig. 14, which is not clear enough. In Fig. 14, however, the authors showed that the behavior of reinforcement in compression would be different than in tension, which is accepted, but where is the model being used?

6. In the paper, the length of plastic hinge was determined based on the reinforcement yielding in compression. The discusser may disagree with this statement, as the reinforcement in a reinforced concrete element under cyclic loading experiences mainly yielding in tension, and yielding in compression may or may not occur. This will depend on many factors, including the location of strain gauge, the intensity of inelastic excursions, the confinement provided, and the concrete strength.[28] Hence, the use of compressive strain may not be a good indicator to estimate the damage under cyclic loading.

7. The research concerned itself with the undamaged length due to the stub effect and it was assumed to be $0.25h$. It is clear that this ratio, $0.25h$, is an assumption and it may exaggerate the undamaged length. The stub effect is not a uniform distance and may take a pyramid shape with a maximum depth at the center and zero at the outer edges. Also, the stub effect

Table B—Validation of proposed method

Specimen	P/P_o	L/h	Measured	Predicted (Eq. (11))	Measured/ predicted
S24-2UT	0.5	5	$0.66h$	$0.69h$	0.96
S17-3UT	0.5	7	$0.91h$	$0.85h$	1.06
S24-4UT	0.2	5	$0.49h$	$0.25h$	**1.96**
S24-5UT	0.2	5	$0.47h$	$0.25h$	**1.88**
Average					1.47

is neither quantifiable nor dependable.[11] Thus, it is preferred to include the stub effect into the analytical hinge length.

8. In Fig. 8, the authors used a strain value of 0.002 to limit the usable compressive strain in the analysis. Such a value may not be correct for different reasons: a) it is smaller than the specified code's value (for example, ACI recommends 0.003), and 2) the section may experience this strain value during the ascending branch, which occurs before reaching the sectional ultimate capacity, as discussed previously. In other words, the hinge will form at a concrete strain value greater than 0.002.

9. The discusser disagrees with the result of Fig. 12(a), which shows that the curvature ductility increases as the reinforcement ratio increases. Although the target of the paper was not evaluating the curvature ductility, the figure implies that the results are not correct and this also contradicts with previous research.[29-32] Because ductility decreases with the reinforcement ratio increase, it is expected that the hinge length decreases as the reinforcement ratio increases.

10. The authors claimed that the proposed method gives good results if compared to the experimental work. As can be seen in Table B, the proposed method failed to estimate the hinge length of the Specimens S24-4UT and S24-5UT. The general, averaged valued is biased, and the analytical method underestimates the experimental values by an average value of 47%.

11. The paper has some editorial errors such as the SI units of reinforcement yield strength, and in Eq. (2), the tip displacement should be replaced by flexural displacement.

REFERENCES

27. CEB, "RC Frames under Earthquake Loading: State of the Art Report," Thomas Telford, 1996, 303 pp.

28. Elmenshawi, A., "Ductility of Ultra-High Strength Concrete Flexural Elements Subjected to Seismic Shear," PhD thesis, Department of Civil Engineering, University of Calgary, AB, Canada, 2008, 360 pp.

29. Tognon, G.; Ursella, G.; and Coppetti, G., "Design and Properties of Concretes with Strength Over 1500 Kgf/cm^2," ACI JOURNAL, *Proceedings* V. 77, No. 3, May-June 1980, pp. 171-178.

30. Park, R., and Ruitong, D., "Ductility of Doubly Reinforced Concrete Beam Sections," *ACI Structural Journal*, V. 85, No. 2, Mar.-Apr. 1988, pp. 217-225.

31. Ho, J.; Kwan, A.; and Pam, H., "Theoretical Analysis of Post-Peak Flexural Behavior of Normal and High Strength Concrete Beams," *The Structural Design of Tall and Special Buildings*, V. 12, 2003, pp. 109-125.

32. Elmenshawi, A., and Brown, T., "Identification of Parameters Affecting Ductility of Reinforced Concrete Beams," *1st International Structural Specialty Conference*, CSCE, Calgary, AB, Canada, ST-009-1:10, 2006.

AUTHORS' CLOSURE

The authors would like to thank the discussers for their comments. The comments made or the questions raised by discusser Subramanian are addressed first.

The authors agree that the term, "measured plastic hinge length," was not well described. Figure 8 provides a comparison between the visually-observed plastic hinge lengths and estimated plastic hinge lengths. The measured plastic hinge lengths are obtained from the measured curvature and tip displacements and by using Eq. (2). As such, they are perhaps better defined as "calculated plastic hinge lengths inferred from measured tip displacements and curvatures." Such an explanation was included in the original submission but inadvertently left out in the final submission of our paper. Nevertheless, the results are illustrated in Table C. A detailed description of the measured plastic hinge lengths and observed plastic hinge lengths were provided in an earlier paper.[33]

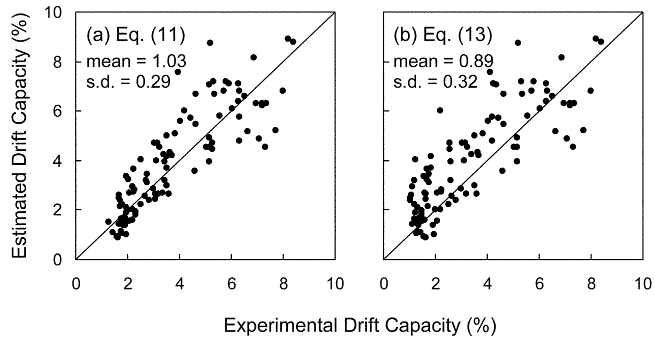

Fig. A—Comparison of plastic hinge lengths.

The plastic hinge lengths suggested by Paulay and Priestley[8] is calculated using Eq. (10) given below.

$$
\begin{aligned}
l_p &= 0.08L + 0.15 d_b f_y \quad (f_y \text{ in ksi}) \\
l_p &= 0.08L + 0.022 d_b f_y \quad (f_y \text{ in MPa})
\end{aligned} \tag{10}
$$

The plastic hinge lengths of the tested specimens using the above expression are re-examined, as shown in Table D. The diameters and yield strength of reinforcing bars are given in Table 1. These values are consistent with those in the original paper.

It is an interesting idea to combine Eq. (11) with an "adjustment factor" for concrete strength, which was developed by Baker.[1] To evaluate different plastic hinge length expressions, the drift capacities of columns are estimated and compared with experimentally measured drift capacities using the column database as illustrated in the paper. The results included in Fig. A indicate that there is no advantage in combining two equations. This is due to the fact that the adjustment factor was developed based on a different test program and, therefore, cannot be simply combined with the plastic hinge length expression developed in this research study.

Most, if not all, of the technical points raised by discusser Elmenshawi are incorrect. The issues raised by this discusser are addressed next:

1. Only some of the early research, which was used for comparison purposes, is based on reinforced concrete members tested under monotonic loads. Most of recent research discussed in the paper is based on reinforced concrete columns tested under reversed cyclic loads. In deriving the proposed plastic hinge length expression, sensitivity analyses were conducted using 121 columns from the UW/PEER column database, all of which were columns subjected to reversed cyclic loads. The authors agree that the depth of the neutral axis may not be important in determining the plastic hinge length. The depth of the section (h or d) is considered as one of the important parameters by many researchers.[1-6,8-10,14] As indicted in the proposed plastic hinge expression (Eq. (11)), the authors considered the depth of the section as an important parameter in terms of the shear span-depth ratio (L/h). The reinforcement ratio was also considered as one of important parameters by other researchers including Park et al.[6] and Priestley and Park.[7]

Table C—Measured plastic hinge length: experiments and estimations[19,33]

Specimen	Cycle no.	Equivalent plastic hinge length		P/P_o	L/h	Photograph
		l_p/h	Average			
S24-2UT	14	0.64	0.66	0.5	5	
	17	0.66				
	20	0.70				
	23	0.68				
	26	0.61				
S17-3UT	14	1.02	0.91	0.5	7	
	17	0.89				
	20	0.82				
	23	0.80				
S24-4UT	14	0.57	0.49	0.2	5	
	17	0.52				
	20	0.50				
	23	0.43				
	26	0.44				
S24-5UT	14	0.50	0.47	0.2	5	
	17	0.50				
	20	0.44				
	23	0.41				

Table D—Predicted hinge lengths using Eq. (10)

	L, in.	d_b, in.	f_y, ksi	l_p
S24-2UT	120	0.875 (No. 7)	73	19.2 in. (= 0.80h)
S17-3UT	120	0.625 (No. 5)	72	16.4 in. (= 0.95h)
S24-4UT	120	0.875 (No. 7)	58	17.2 in. (= 0.72h)
S24-5UT	120	0.625 (No. 5)	58	17.2 in. (= 0.72h)

2. The plastic hinge length is used to relate curvatures of a column to displacements when a column undergoes inelastic deformations. Therefore, the discussion of plastic hinge lengths is more meaningful for moderately ductile or ductile columns. When column behavior is significantly affected by large shear stresses so that shear failure is expected, the behavior of such columns will be brittle. As a result, the plastic hinge length will not be of interest for such columns. On this basis, the columns with diagonal shear cracks were not considered in the paper, as indicated in the paper. The effect of shear span-depth ratio on plastic hinge lengths was also examined in the sensitivity analyses using the test results from columns tested under reversed cyclic loads. This was supported by many researchers for monotonically-loaded[1-5,10] and cyclically-loaded[6-8,14] specimens.

The authors believe that with respect to the length of a plastic hinge, the fixed-end rotation is not important. As stated previously, the plastic hinge length is meaningful when the concrete members are in the postpeak part of their response. The fixed-end rotation will simply add to the tip displacement but will not influence the plastic hinge length.

3. Irrespective of loading conditions (monotonic or cyclic loading), plastic hinges form in columns that have moderately ductile or ductile behavior. Both for monotonic and reversed cyclic loading, severely damaged regions of columns will experience buckling of reinforcing bars under compression and yielding of bars under tension. Previous research on bar buckling[34] shows that bar buckling initiates at the yield strain for most cases where the unsupported longitudinal bar length-to-longitudinal bar diameter ratio is less than 10. Based on this observation, Step 4 of the proposed concrete compression strain method estimates the plastic hinge length as the length of the region in which reinforcing bar strains are larger than the yield strain. Also, previous experimental and analytical studies on reinforced concrete columns show that the backbone curves obtained from cyclic tests generally agree with the column response obtained from monotonic tests.

4. As illustrated in our paper and in Item 3 above, the reinforcing bar buckling is considered in the compression strain method. In addition, this reinforcing bar buckling is taken into account in generating the moment-curvature relationship in Step 1 of the proposed method.

5. The detailed explanation of the material modeling in generating the moment-curvature relationship was not provided in our paper for the sake of brevity. A comprehensive discussion can be found in elsewhere.[19]

6. As discussed in Item 3, the yield strain is used to consider both bar yielding under tension and bar buckling under compression. The confinement and concrete strength may have some influence on plastic hinge lengths. However, as long as columns are moderately or highly ductile, which means more confinement for higher concrete strengths, it is the authors' opinion that the effect of these parameters is insignificant in estimating the plastic hinge lengths.

7. The direct inclusion of stub confinement into the analysis of column behavior will not be possible unless a very sophisticated finite element model is used. Such sophistication is not warranted to achieve the primary objectives listed in our paper. The superior performance of the proposed method is demonstrated on the UW/PEER database where stub confinement was ignored.

8. The use of yield strain of 0.002 is not to limit the usable compressive strain in the analysis and is not relevant to the concrete ultimate strain. As discussed in Item 3, it is used to estimate the length of the severely damaged column region by considering the reinforcing bar yielding and buckling.

9. Figure 12(a) does not show an increase of curvature ductility with the increase of longitudinal reinforcement ratio. Because the curvature ductility is the ratio of the ultimate curvature to the yield curvature, the increase of the ultimate curvature does not mean the increase of the curvature ductility.

10. As explained in the response to the discussion by Subramanian, the measured plastic hinge lengths were obtained using Eq. (2) with measured curvatures and displacements. As such, the measured plastic hinge lengths relate the curvature to the tip displacement. On the other hand, the proposed plastic hinge length is based on the relation between the curvature and the flexural deformations. Therefore, there is a subtle but important conceptual difference between the measured and proposed plastic hinge lengths. As clearly stated in our paper, deformations due to shear stresses and bar slip have to be explicitly considered and added to the flexural deformation component when the proposed plastic hinge length is used. Figure 16 confirms that accurate prediction of column member responses for S24-4UT and S24-5UT can be obtained by both methods.

11. The editorial errors in the conversions to SI units for reinforcement yield stresses in Table 1 are noted. The tip displacement is the right-hand term in Eq. (2), as it includes shear and bar slip displacement components.

REFERENCES

33. Bae, S., and Bayrak, O., "Seismic Performance of Full-Scale Reinforced Concrete Columns," *ACI Structural Journal*, V. 105, No. 2, Mar.-Apr. 2008, pp. 123-133.

34. Bae, S.; Mieses, A. M.; and Bayrak, O., "Inelastic Buckling of Reinforcing Bars," *Journal of Structural Engineering*, ASCE, V. 131, No. 2, Feb. 2005, pp. 314-321.

Disc. 105-S29/From the May-June 2008 *ACI Structural Journal*, p. 301

Control of Flexural Cracking in Reinforced Concrete. Paper by R. Ian Gilbert

Discussion by Andor Windisch

ACI member, PhD, Karlsfled, Germany

The author argues against "overly simplistic methods" and promises a rational one for flexural cracking control, especially emphasizing the impact of shrinkage. The discusser agrees but does not find a more exact formulation in the paper.

In the introduction, early cracking caused by the internal restraining tensile force and in restrained structures due to shrinkage are mentioned. The paper itself does not address these important issues.

In Fig. 1 and 2 and Eq. (6) of the paper, the author implies that the crack width depends on the crack spacing—in effect, to the width of a given crack those steel and concrete strains contribute, which develop on both sides of this crack over the length where slip between steel and concrete occur. The crack width calculated according to Fig. 2 can develop only if both neighboring cracks are at the same distance s. Additionally, it is not helpful that the notation is not precise; for example, there are two "stresses in tensile steel at crack"—σ_{sr1} and f_{st}, respectively.

The comparison with test data reveals that the ratio w^*/w_{max} (that is, predicted versus maximum measured crack widths) shows systematic deviations: in the case of beams at low steel stresses (150 N/mm^2 [21.75 ksi]), it strongly overestimates, but at higher steel stresses (220 N/mm^2 [31.9 ksi]) underestimates, the crack width. In the case of slabs, the extremely strong influence of the reinforcing bar spacing can be found, which cannot be predicted by the model. It is state of the art that the concrete cover strongly influences the crack width, too. This influence is missing in the proposed model.

No information or guidance is provided for the designer regarding the amount of shrinkage that should be taken into account; in the tests, very fresh concrete members were subjected to constant sustained loading. In a real structural member, the loading resulting in cracking might occur after the bulk of shrinkage had already developed.

Crack control is an important step of the design and may have a significant impact on the economic efficiency of concrete structures. The author is encouraged to look for enough simple but physically more reliable models.

Disc. 105-S30/From the May-June 2008 *ACI Structural Journal*, p. 308

Shear Strength of Thin-Webbed Post-Tensioned Beams. Paper by Miguel Fernández Ruiz and Aurelio Muttoni

Discussion by Andor Windisch

ACI member, PhD, Karlsfled, Germany

The authors report on tests on thin-webbed, post-tensioned beams failing by crushing of the web and analyze their load-carrying mechanism using different stress fields of increasing complexity.

ANALYSIS USING STRAIGHT STRESS FIELDS

The derivation of the compressive field angle θ and concrete stresses in the web σ_c must be discussed:

1. Regarding Eq. (2) and (3), the unknown quantities at design are ρ_w, θ, and σ_c, and at control of the load-carrying capacity, V, θ, and σ_c. How can three unknown quantities be determined from two equations?

2. Why was the angle θ deduced from Eq. (2)? According to Fig. 5, SH3 failed due to web crushing. This means: $\sigma_c = f_{ce}$; that is, θ should have been calculated from Eq. (3).

3. Why is the lever arm chosen just z = 1100 mm (43.3 in). How is it compatible with the flexural equilibrium at midspan?

4. Why was no fan-out applied at load transfer of the horizontal normal force N? It violates De Saint-Venant's principle, as well. And yet, under the vertical load V, a pronounced fan-out is shown (refer to Fig. 13).

5. Why does the fan-out of the vertical load in the straight stress field differ from that at the discontinuous stress field analysis?

6. How was the influence of the pretensioned wires considered for the determination of the compression field angle? A difference must exist whether the beam section is pretensioned or not. In the equilibrium considerations shown in Fig. 11(b), neither C nor T contains the residual prestressing force of approximately 0.245 MN (54.75 kips).

7. Upon calculating the value of θ, only the influence of the vertical component of the PT tendon can be perceived. How are the horizontal component of the PT tendon and the influence of the external axial load N taken into account?

8. The equilibrium of compression field given in Fig. 12(b) is not valid for the straight stress field shown in Fig. 11(b)—no parallel stress field with the width of $z^* \cos\theta$ can be found there.

9. The failure patterns of the Specimens SH1 and SH3 shown in Fig. 5 reveal another problem: between the loading point of V and the curved prestressing tendons, the web must transfer the entire inclined compressive force. The shear contribution of the inclined prestressing tendons acts/unloads the web "below" the tendons only. This remains true even if one obstinately adheres to the parallel chorded truss and operates with geometrical interpretation (very flat struts) instead of considering mechanically correct components of the composite material structural concrete.

10. The 20-degree angle calculated for the compression strut contradicts the crack pattern of SH3 shown in Fig. 5 and the computed principal compressive strain directions given in Fig. 9(b). The plot of midspan deflection versus applied load (Fig. 6) reveals that during the first two loading phases that increasing V up to 0.81 MN (181 kips) (without applying the external axial force N) achieved very pronounced plastic deformations, that is, a pronounced crack pattern. During application of the axial load, which was increased thereafter overproportional to the vertical load, the beam behaved much stiffer. Due to this external prestressing force N, a different crack pattern developed. This unusual loading history must be kept in mind while looking at the cracking pattern shown in Fig. 5. Any compliance of the compressive field angle could occur by chance only.

DEVIATED STRESS FIELD

The authors stated that as "a straight stress field does not allow one to increase the force in the prestressing tendon" the equilibrium of moment at midspan can not be satisfied: a "more consistent stress field" must be chosen. Thus it must be concluded that straight stress fields can not be applied for the analysis of PT beams. A further question arises: how can straight stress fields be applied to ordinary reinforced concrete structures when at midspan, the yielding of the ordinary reinforcement can not be automatically assumed. Why should be any difference in the respective increases of stresses in ordinary and bonded high-strength steels? The authors reported about 11 mm (0.433 in.) wide cracks at midspan. The increase of the force in the PT tendon at midspan was not influenced, whether or not straight of deviated stress fields were considered.

In the critical zone of the stress field, as shown in Fig. 13(c), the free body is in the middle. What would the critical zone look like if the loaded span a_N had been the same length as this free body was? In this case, the critical free body seems to disappear.

DISCUSSION ON SUITABILITY OF DISCONTINUOUS STRESS FIELDS

Shear strength

The shear strength of the web, f_{ce}, according to the stress field method, is a rather arbitrary quantity that seems small enough:

1. As the literature[9] was not available for the discusser, some clarifications would have been useful to learn why η_{fc} was introduced and why only 30 N/mm (4350 psi) was chosen as reference strength.

2. The same quandaries in Item 1 hold true for η_ε.

3. Furthermore, there is another geometrical influence that should be considered in η_D: the stirrups and longitudinal reinforcing bars crossing cracks sustain slip to concrete; this results in some slight voids between the reinforcing bar and concrete that must be considered similarly in η_D, like in case of the ducts.

4. The concrete spalling after failure, shown in Fig. 5, occurs as follows: the reported vertical strains larger than 5‰ result in cracks strictly inclined to the reinforcing bars. The reinforcing bars buckle, which causes the concrete cover spall; the remaining web cross section between the reinforcing bars becomes overstressed and fails: the failure pattern is shown in Fig. 5.

CONTINUOUS STRESS FIELD

The authors are correct that the most compatible stress fields are those that present a smooth variation of the stresses between adjacent struts. For their development, the authors make use of FEM where the different influencing factors are split: the influence of the tendons in the web is considered as effective width, and the concrete strength reduction factor is introduced locally for each element, for example, according to Vecchio and Collins.[14] Some remarks:

1. It is highly appreciated that the geometrical and material characteristics are modeled separately.

2. The discusser showed in a paper[15] that the η_ε factors of Vecchio and Collins do not model any strength reduction of concrete. The failure of the "Toronto" panels, which were loaded by controlled deformations, was introduced by the yielding of the weaker course of reinforcement. This could be one reason why the FE model in the paper overestimated the actual strength of Beam SH3.

3. The good agreement of the minimum angle of 19.6 degrees with the discontinuous stress field results is rather irrelevant as the failure was located by the FEM outside the fan region.

4. The strain reduction factor η_ε found in the critical zone (nonetheless, outside the fan region, where it should be valid) confirmed the design values usually adopted for this parameter, but no relevance of η_{fc} was detected.

5. The authors and Fig. 15(d) reveal the important participation of the upper flange carrying the shear forces even in the critical regions. It is necessary to reintegrate the upper flange/compressive zone into our future shear models.

SHEAR STRENGTH BASED ON STRESS FIELDS AND COMPARISON WITH CODES OF PRACTICE

Some comments:

1. Each angle θ found in the three different stress fields is smaller than the limits given in the codes. The angle determined in the most detailed model, the continuous stress field, differs the most.

2. ACI 318-05 "seems to be here the most reactionary." Nevertheless, if ACI 318 manages to incorporate the proper contribution of the compressive zone into its shear calculation model, then it can get ahead of all of the "up-to-date" models.

3. The continuous stress field model provided the best agreement with the test results, but: a) the way and place of failure was different to the test; and b) the tedious model can not be used for practical purposes.

CONCLUSIONS

Comments on the conclusions from the tests:

1. It is obvious that the strain increases in bonded tendons during the loading. The conclusion must be that straight stress fields cannot be used in case of structural members with bonded prestressed tendons;

2. It is trivial that bending-shear cracks become flatter approaching the compressive zone. As the compression field model never addressed the crack direction, it cannot be used as proof; and

3. The non-negligible influence of the compression chord found in the experiments should lead to the reason that shear models not considering the participation of the compression chord should be disqualified in the future.

Comments to the conclusions from the theoretical studies:

1. Multiplicative approach for the strength of the web is acceptable as it smooths the uneconomical effects resulting from the uncertainty in our knowledge concerning the effect of deviations in the strength reductions;

2. The deviation of the stress field is a result of the increase of stresses, not the cause;

3. The conservatism of the results of discontinuous stress fields is due to the deficiency of this model. The contribution of the compression chord cannot, and should not, be modeled with any further flattening of the stress fields;

4. The use of continuous stress fields is not practical. Angles close to 20 degrees for the compression field in the web is beyond any limits given in SIA and AASTHO; hence, this model is irrelevant;

5. The continuous stress field revealed the importance of the compression chord for the shear strength. Eurocode 2, SIA 262, and AASHTO LRFD strive for a mechanically sound design model, which is certainly not a parallel-chord or flat-angle truss model without an additive term considering the contribution of the compression chord. The absence of a compression chord makes the model mechanically questionable; and

6. The application of an increasingly complicated calculation method to achieve the "sufficient strength result" is not practical: the owner is always interested in the maximum load-carrying capacity of his/her structure. Moreover, with increasing "accuracy," that is, more and more laborious models and higher and higher concrete stresses in the fan-out regions, the increasing efforts actually decrease the calculated safety factors of the structure.

The paper gives very valuable insights for sound mechanical models on shear strength.

REFERENCES

15. Windisch, A., "On the Design of Two-Way Reinforcements in R/C," Studi e ricerche, Scuola di specializzazione in Costruzioni in C. A. Fratelli Pesenti, Politecnico di Milano, V. 21, 2000, pp. 283-302.

AUTHORS' CLOSURE

The authors would like to thank the discusser for his interest in the paper, for his positive perspectives on the stress field method, and for providing the authors an opportunity to clarify some issues of the paper.

Section "Analysis using straight stress fields"

1. In fact, there are only two unknowns. Either ρ_w is known (assessment of an existing structure) or θ is set to a design value (design of new structures).

2. The purpose was to compare the values of the stresses in the compression field obtained by using various approaches and to compare them to the estimated strength. The analyses respecting the governing failure mode are given in Table 4.

3. It was chosen as the distance between centers of gravity of the upper and lower chords (accounting for the latter the prestressing tendons layout).

4. As the discusser states, the horizontal force *N* spreads at the load-introduction region according to the Saint-Venant principle (this topic has been investigated by the authors for prestressed beams in Muttoni et al.[9]). However, such consideration was not included in the paper because the region between the introduction of the horizontal force *N* and the support is not governing at ultimate.

5. The question is unclear to the authors. We think the discusser refers to "deviated" where he writes "discontinuous." In that case, the difference is due to the increase in the stress of the tendon.

6. The residual prestressing force cannot be activated on the chosen straight stress field, as the compression field in the web is not deviated. On the contrary, it is activated on the deviated and on the continuous stress fields.

7. Due to compatibility conditions, it is clear that, in the presence of significant axial compression forces, flatter angles of the compression field in the web can be activated. As the discontinuous stress field method is only based on equilibrium conditions, this fact should be taken into account by the designer with additional considerations (for instance, by developing a continuous stress field).

8. At stirrup yielding, Eq. (2) is still valid in the critical strut (outside the fan region). The discusser can easily work it by isolating a single strut or a compression field of differential width.

9. In fact, according to Fig. 9(b), significant compressive strains were measured in both regions over and below the tendons. This is consistent with the chosen stress field (refer to Fig. A(c)). Other equilibrium solutions (Fig. A(b)) were presented and discussed in Muttoni et al.[9] and were considered at the time the paper was prepared. However, they were not found to be suitable and efficient, as the available reinforcement in the tension flange was not used (although it was activated due to flexural cracking). Combinations of both load-carrying mechanisms can still be considered to maximize the failure load (what is in fact done by using the continuous stress field method).

10. The authors are not in agreement with the statement of the discusser. The measured directions of the principal compressive strains were around 20 degrees consistently for the various tests (refer to Fig. 14 and Reference 6). The loading history was, in fact, more complicated, as the beams were extracted from an actual bridge (in service for 36 years). This is why the stress field results are compared to the measured principal strain directions and not to the cracking pattern (Fig. 14).

Section "Deviated stress field"

The authors disagree with the opinion of the discusser. Although a straight stress field can be improved, it is still a simple and effective tool to estimate the strength of a member. Furthermore, straight stress fields are proposed in most codes of practice as they provide safe estimates and constitute a good compromise between time devoted to analysis and accuracy. With respect to the shape of the critical zone in case the shear span is shorter, the critical zone will no longer be the same, as direct strutting of the load will be possible (and thus a different load-carrying mechanism develops).

Section "Discussion on suitability of discontinuous stress fields"

1. The discusser should note that the reference is a book published in 1997 by Birkhäuser (distributed worldwide and available in English and German) and it should not be very difficult to find copies of it. The value the discusser refers to (30 MPa [4350 psi]) was obtained by comparisons of the stress field method to test results.

2. The cited reference by Vecchio and Collins[14] covers this topic.

3. The authors agree with the discusser. In fact, when large-diameter bars (with respect to the web thickness) are used, the reduction of η_D could be applicable. Such cases, however, are rarely found in practice.

4. The explanation given by the discusser seems reasonable and in agreement with previous works by Muttoni et al.[9] (refer to Fig. 3.12 of that reference).

Section "Continuous stress field"

2. This question is not replied to as it requires discussing works outside the paper.

3. In the authors' opinion, it seems highly relevant, as failure occurred outside the fan region (as predicted by all stress field analyses).

4. The values of the estimated strength accounted for the strength reduction factor η_{fc}, which plays a significant role.

5. The authors agree with the opinion of the discusser. Due to the required amount of work, however, it seems only reasonable for special cases (when a strengthening can be avoided, for instance).

Section "Shear strength based on stress fields and comparison with codes of practice"

1. It should be noted that the values recommended in codes of practice are given for design of new structures as conservative and generally-applicable rules.

2. In the authors' opinion, ACI is not "reactionary." Simply, it is a code for structural concrete and not fully adapted to prestressed members. Instead of introducing the contribution of the compression chord, it seems more pertinent in the authors' opinion to deal with the influence of prestressing and variable compression field angle in a more consistent way.

3. In fact, the way and place of failure were exactly those recorded at testing.

With respect to its use for practical purposes, the authors agree that it is not suitable for day-to-day applications. However, it is a very valuable tool in assessing the strength of an existing structure in order to avoid an expensive strengthening of a structure.

Section "Conclusions from tests"

1. The authors disagree with this statement. Although they can be improved, they are highly recommended to perform a safe design of a new structure within a reasonable time (straight stress fields are thus proposed in most codes of practice).

2. The question is unclear to the authors.

3. The authors disagree that such models should be disqualified. On the contrary, they allow safe and accurate estimates of the strength with a limited amount of work, which is normally sufficient in practice.

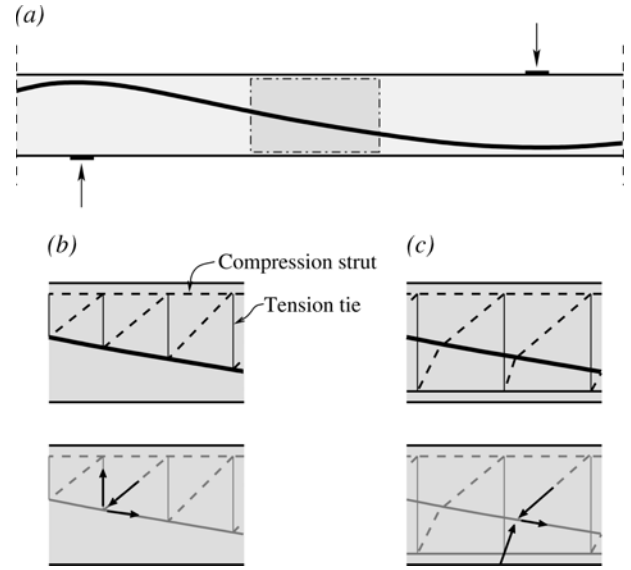

Fig. A—Load-carrying mechanisms in post-tensioned beams: (a) specimens and detail of investigated region; (b) load-carrying mechanism accounting only for prestressing tendon; and (c) load-carrying mechanism accounting for tendon and reinforcement in tension flange.

Section "Conclusions from theoretical studies"

1. The authors think it is a safe and reasonable approach, leading to good correlation with test results.

2. Both phenomena are necessary to satisfy equilibrium. Whether the discusser prefers to think that one is the cause or the effect does not seem relevant.

3. "Deficiency" seems a negative word. The authors think that the conservatism is due to the set of safe and simple hypotheses adopted. Nevertheless, the results obtained are rather accurate and the conservative hypotheses can be refined step-by-step.

4. Developing angles of the compression field beyond the limits given in the Swiss or American Code does not seem to make such models "irrelevant." On the contrary, codes are written to provide safe provisions for design that are generally applicable. However, the limits given in such codes can be overcome in many cases, for instance, when the strength of existing structures is assessed.

5. The authors think it is hard to justify that statement except for special cases.

6. The authors mostly disagree with this statement. The owner is normally interested in a safe and economical structure for the foreseen use of the structure (its "maximum load-carrying capacity" is normally irrelevant for the owner in the authors' experience). Safety margins for the models of 20% (straight stress fields) or 10% (deviated stress fields) are very reasonable if compared to the safety margins adopted for other models, such as punching shear, for instance.

The last remark of the discusser about decreasing the "calculated safety of the structure" when the accuracy is increased is contradictory with his previous remark that "the client is always interested in the maximum load-carrying capacity," and thus that only refined and accurate models should be used. In the authors' opinion, for design of new structures, it is normally sufficient to perform a straight stress field and, in some cases, a deviated one. For assessing an existing structure, the accuracy can be refined if necessary to avoid, if possible, an expensive strengthening.